# Yet Another Introduction to Quantum Computing

\*I will simply express my strong belief, that that point of self-education which consists in teaching the mind to resist its desires and inclinations, until they are proved to be right, is the most important of all, not only in things of natural philosophy, but in every department of dally life.\* Michael Faraday (1859) [1]

This jupyter book (to be compiled) is a collection of notebooks introducing the fundamentals of quantum computation, including a preliminary introduction to quantum mechanics. It is assumed that the reader has some high school level of mathematics. A basic understanding of trigonometry is important. To the extent that it is practical, the mathematics needed is covered in Chapter 3.

# 0. Recommended Reading

Rather than \*standing on the shoulders of giants\* this work sits on a pile of textbooks, YouTube channels and online resources. In particular, a reader may benefit from the following materials:

Great introduction materials:

- [Introduction to Classical and Quantum Computing, Thomas G Wong](http://www.thomaswong.net/introduction-to-classical-and-quantum-computing-1e2p.pdf)

- Quantum Mechanics: The Theoretical minimum, Leonard Susskind & Art Friedman

- Q is for Quantum, Terry Rudolph

For more coding based understanding of QC:

- [The Qiskit Textbook](https://qiskit.org/textbook/preface.html)

- [Qiskit YouTube channel](https://www.youtube.com/c/qiskit)

- [Xanadu Quantum Codebok](https://codebook.xanadu.ai/)

For those with more experience of physics and maths (undergraduate level)

- Quantum Computation and Quantum Information, Nielsen & Chuang

- Introduction to Quantum Mechanics, David J Griffiths

In addition, each chapter may have some references with relevant links.

## 1. What is Quantum?

### 1.1 Common misconceptions about quantum

In science fiction the adjective quantum is often used to explain how a technology that seems to defy the laws of physics works. In the 2018 superhero movie \*Ant-Man\*, the protagonist is able to access this realm by becoming smaller and smaller until they enter the mysterious \*Quantum Realm\*, described as “a reality where all concepts of time and space become irrelevant”.

If this was how subatomic physics worked, there would be no subatomic physics.

Quantum mechanics is described by the Schrödinger equation that is built on space and time. Quantum physics has some differences to classical physics, the physics everyone is used to experiencing in everyday life. Quantum it is not some magical concept which allows for anything that violates classical physics to be explained away. There are however, some small but significant quirks of quantum mechanics that allow for the development of new technologies.

### 1.2 How many physicists does it take to change a lightbulb?

In the early 20th century, a German physicist (by the cool name of Max Planck) was tasked with modelling the emission of light from a filament bulb (basically a wire that gets hot enough to glow). He came up with what is known as the black body radiation spectrum. You may be wondering why such trivia has been included in a textbook about quantum computing. It turns out, from his model, the energy emitted by an object with a finite temperature is not emitted in a continuous spectrum. That's to say rather than all possible wavelengths of light being emitted by this bulb only certain wavelengths were allowed. This is known as a discrete spectrum. What this in turn means is that energy is emitted are chunks, or \*quanta\*, with a well-defined energy. We can't have energy between these chunks. It's a bit like taking the lift (elevator in American English). One can only get on or off the lift at specific heights corresponding to the floor level. Each floor is seperated by a distance and the lift can go up or down in any multiple of these distances limited by the number of floors. In between those levels the doors of the lift won't open.

This concept of quantisation of energy is where the quantum comes from. Whilst this concept is less significant to quantum computing, it is still worth mentioning. The reason for this is that the transistor that powers all digital electronics is built upon this concept. Thus all computations using this technology (the integrated circuit) are computers running on quantum mechanics. Which is not to say they are quantum computers simply that they are fundamentally built upon quantum physics, as are you, I, and everything we can see and touch.

Beyond the quantisation of energy, there are quantum phenomena which are very much more significant to quantum computing: \*superposition & entanglement\*.

### 1.3 So what happened to Schrödinger’s cat?

You may have heard of a very special cat associated with another German physicist, Erwin Schrödinger. In this analogy, a cat is kept inside a black box that the observer can't view inside. Within the box, there is also a vial of poison with a radioactive source. The radioactive source has an equal probability of decaying or not decaying in a particular time interval. If the source decays, the poison is released and the cat dies. otherwise the cat is just fine. Until the observer opens the box they don't know if the cat is dead or alive. Schrodinger came up with this analogy to explain why quantum mechanics was so controversial for physicists that had been able to do very well with classical physics until the turn of the 20th century. This analogy is effective but is far removed from the everyday experiences one has, so another analogy can be made that encapsulates the same physics.

\*It is as simple as tossing a coin. \*

A normal coin has two faces: heads & tails. Assuming the coin is unbiased, When the coin is tossed there should be an equal probability of landing on a heads, $P(H)$ or a tails, $P(T)$, both equal to $1/2$. Here the extremely unlikely case where the coin lands on its round edge is not really considered. When the coin has not yet landed, we can say it is in a particular state where it is \*like being both\* heads and tails at the same time. That's not to say there is 1 coin with a heads and another coin that is a tails\*, but more that there is some uncertainty in the state of the coin.

In quantum mechanics we call this particular state a \*superposition\*. And this is the state we say the Schrodinger's cat is in before the box is opened. The concept of looking at the coin, or opening the box, is what in quantum mechanics is known as \*performing a measurement\*.

$\*$ According to the many worlds interpretation of quantum mechanics it is not quite so simple...

### 1.4 Not so spooky action at a distance

The more conceptually difficult phenomena quintessential to quantum computing can also be explained using coins (but with one small modification). For this example 2 coins will be required.

Tossing two unbiased coins is effectively the same as tossing the same coin twice and recording the outcome each time. Guessing whether the coin is H or T is 50% for each coin or 25% for getting both of them correct.

For the next step some removeable adhesive will be required. Imagine placing the coins together with each coin having the H facing outwards and the T's stuck together to make one coin twice as thick. Tossing this coin will always result in one coin with the H facing upwards. But before the coin is revealed, the two coins are separated. The coin at the bottom must have its T side up, leaving 2 coins one H and the other T. After shuffling the coins they are separated.

![Unit\_circle](Images\Entanglement.png)

[2]

At this point, guessing either of the coin would have a 50% chance of getting the right outcome- the same as for two coins tossed separately.

But if only one of the coins is revealed, say the H coin, then it is instantly known that the other one must be a T. Suddenly guessing the state of the second coin has a 100% chance of success whereas with the separate coin tosses revealing one made no difference to guessing the other. These linked probabilities is what is known as an \*entangled state\*.

### 1.5 Quantum Technology

Quantum Technologies, as they are defined by John Morton, director of UCLQ (as of the time of writing) are \*"ones which exploit quantum superposition and entanglement to achieve major advances over current technologies in areas including communication, sensing and information processing."\*

These are devices that take advantage of the aforementioned phenomena for practical applications to do something better than what can be done without using them. Quantum computation is one of the most exciting quantum technologies even if it is much less mature than others like quantum metrology (sensing) and encryption (communications).

# ## 2. What is quantum computing

In the previous chapter, some of the fundamental concepts of quantum mechanics were introduced and their relevance to quantum computing was briefly explained. Building on that, this chapter will provide a bit more detail on how quantum computers employ such esoteric, exotic phenomena for practical applications.

### 2.1 More than flipping coins

The example of coins being flipped can be helpful in explaining how quantum effects behave but it would be rather impractical to use coins as a method of quantum information processing: quantum computation. There are a few obvious reasons for this:

- Flipping coins is not very fast. Conventional computers (CPUs) have a clock speed in the GHz or billions of complete clock cycles per second. We would need an astronomical number of coins to try and replicate that.

- Flipping coins can only really be used to generate random numbers. Beyond the conventional deterministic computing operations we can already do, the only real advantage from quantum theory would be the 'random' nature of the coin flips.

- Sticking coins together is even slower than just flipping them. Not to mention the part where they get shuffled in a pseudorandom way.

In other words, a practical quantum computer would, as one might expect, need to be a quantum system that can be controlled effectively and efficiently to carry out useful computations.

### 2.2 So what do computers do

Before we add quantum spice to our computers, it would be wise to first describe what it is that our classical computers do.

Computers work by executing logical operations (think addition/subtraction/multiplication/...) that take the machine from a starting point to a finishing point. The starting point that is fed into the computer is congenitally known as the input and what the computer ends up with is usually referred to as the output. An \*algorithm\* is a set of instructions that allow the computer to take the input and generate an output. For example, an algorithm for doubling a number could take an input of 3, multiply it by 2 and return an output of 6.

Going back to the coins here, this is a bit like laying them out in a row and then proceeding to shuffle and flip the coins according to a script that allows something to be done. For example, you could add 2 numbers expressed in binary by following a simple series of rules that allows for addition. This is not too dissimilar to using an abacus as a calculator. Since the advent of the digital computer in the 1940's, these operations have been carried out by increasingly advanced machines to greater and greater success.

This description of computation is woefully inaccurate by modern standards, but it suffices to outline the basic fundamentals and is sufficient to motivate the use of quantum systems.

### 2.3 Quantum Algorithms

Conventional computers are really great at what they do. The field of high-performance computing has had many decades to mature. According to Top500, an index tracking the most powerful supercomputers, the most powerful supercomputer, Fugaku is a $1bn powerhouse. It consumes 29,899 kilowatts to power 7,630,848 cores. For context, a mid-range laptop today might have 6 cores and consume a total of around 30 watts of power.

For both machines, they execute algorithms that take an input and generate outputs in a similar manner. Whilst they are both excellent at solving a great many problems, there are two things they can't handle so well: superposition & entanglement.

Starting with a simple coin toss there are 2 possible outcomes. If we add a second coin there are 4 possible outcomes (HH, HT, TH, TT). As more and more coins are added to the tosser, the number of possible outcomes, $N$, grows as $N = 2^n$. If we had 300 coins, that would mean a single coin toss has more possible outcomes than we estimate the total number of atoms in the universe. That's roughly 2 followed by 90 0's different combinations of H & T. Now imagine tossing these coins 300 times...

A quantum computer could handle this problem easily. Instead of needing a universe of atoms, 300 qubits \*of sufficiently high quality\* would be enough to run a quantum algorithm that could simulate these coins (including any fancy sticking of them together) to arbitrary precision.

Quantum algorithms are similar to the input and output of classical algorithms but with the addition of superposition and entanglement in the middle. If an algorithm doesn't feature these two concepts, a classical solution will outperform any quantum computer any time.

### 2.4 Quantum Advantage

If our conventional computers perform so many tasks better, there must be some motivation for quantum computing being such an exciting area of research. Aside from the elegance of manipulating quantum states, quantum computing is expected to be more useful than a glorified coin tossing simulator. Despite the limitations in state-of-the-art quantum computing machines, it is known of a few valuable applications where quantum algorithms \*theoretically outperform\* the best possible classical solution. In these specific applications it is said there is a proven \*quantum advantage\* as using the quantum computer derives some benefit to the user for that application. Whether or not it is worth using a quantum computer for a specific task is a very complicated question. It is not as simple as finding out if a quantum computer is faster but also whether the output of the quantum computer is better than what could have been done with conventional algorithms on a powerful classical computer. As of the time of writing, there has been no practical demonstration of a problem with real world (outside experimental physics) problem where using a quantum computer was better than using the tried and tested classical solution. As quantum computers become increasingly powerful it is hoped that this threshold will be crossed in the next few years...

# ## The most powerful quantum subroutine

\*Mathematical analysis is as extensive as nature itself; it defines all perceptible relations, measures times, spaces, forces, temperatures:;; this difficult science is formed slowly, but it preserves every principle which it has once acquired; it grows and strengthens itself incessantly in the midst of the many variations and errors of the human mind. Its chief attribute is clearness; it has no marks to express confused notations. It brings together phenomena the most diverse and discovers the hidden analogies which unite them. \* - Joseph Fourier, The Analytical Theory of Heat (1878)

This chapter is atypical of the rest of this textbook. Here a rather lengthy historical introduction is given in an attempt to convey the power of what could be considered the most powerful quantum algorithm.

## Signal processing

Waves surround us. Light travels from the sun to our eyes and sound travels from vibrating objects to our ears and those two waves comprise much of our experience of reality. For both light and sound, what we are sensitive to is small changes of intensity, the power per unit area, that the complex network of our sense organs and brain are able to process and interpret as what we see and hear. In engineering, signal processing is extremely important in telecommunications and in physics there are insights we can gain about the nature of solids by performing a similar analysis. In the latter two cases, the mathematics we use to acomplish these tasks is described by various Fourier Transforms.

In 1822, French mathematician, Joseph Fourier proposed a method for